Note Title EE362 Review Session 4 Fall 2018 12/10/2018

EX 1

$$X(z) = \frac{z^2+1}{z^2-1} = \frac{z^2+1}{(z+1)(z-1)}$$

$M = N \Rightarrow$ divide by z

$$\frac{X(z)}{z} = \frac{z^2+1}{z(z+1)(z-1)} = \frac{A}{z} + \frac{B}{z+1} + \frac{C}{z-1}$$

$$A = z\left(\frac{X(z)}{z}\right)\Big|_{z=0} = \frac{z^2+1}{(z+1)(z-1)}\Big|_{z=0} = -1$$

$$B = (z+1)\left(\frac{X(z)}{z}\right)\Big|_{z=-1} = \frac{z^2+1}{z(z-1)}\Big|_{z=-1} = \frac{2}{(-1)(-1-1)} = 1$$

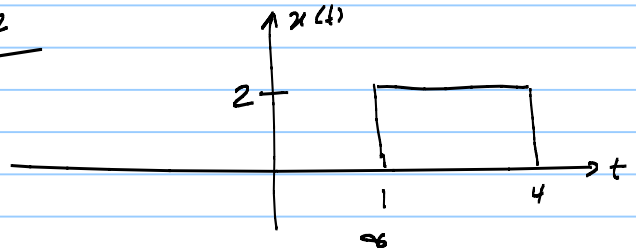
$$C = (z-1)\left(\frac{X(z)}{z}\right)\Big|_{z=1} = \frac{z^2+1}{z(z+1)}\Big|_{z=1} = \frac{2}{(1)(1+1)} = 1$$

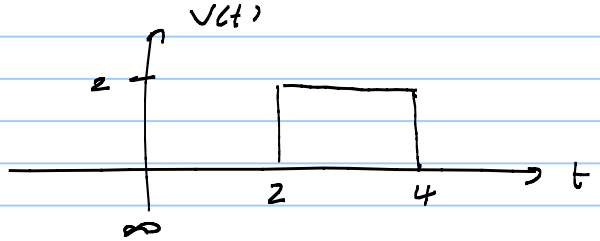
$$\frac{X(z)}{z} = \frac{-1}{z} + \frac{1}{z+1} + \frac{1}{z-1}$$

$$X(z) = -1 + \frac{z}{z+1} + \frac{z}{z-1}$$

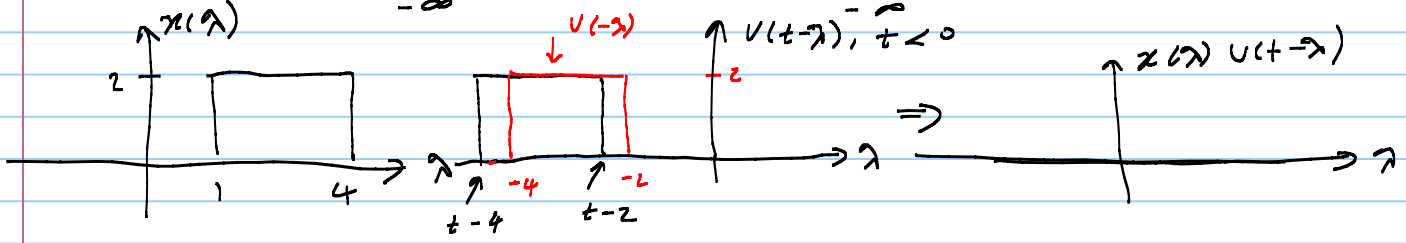
$$x[n] = -\delta[n] + (-1)^n u[n] + (1)^n u[n]$$

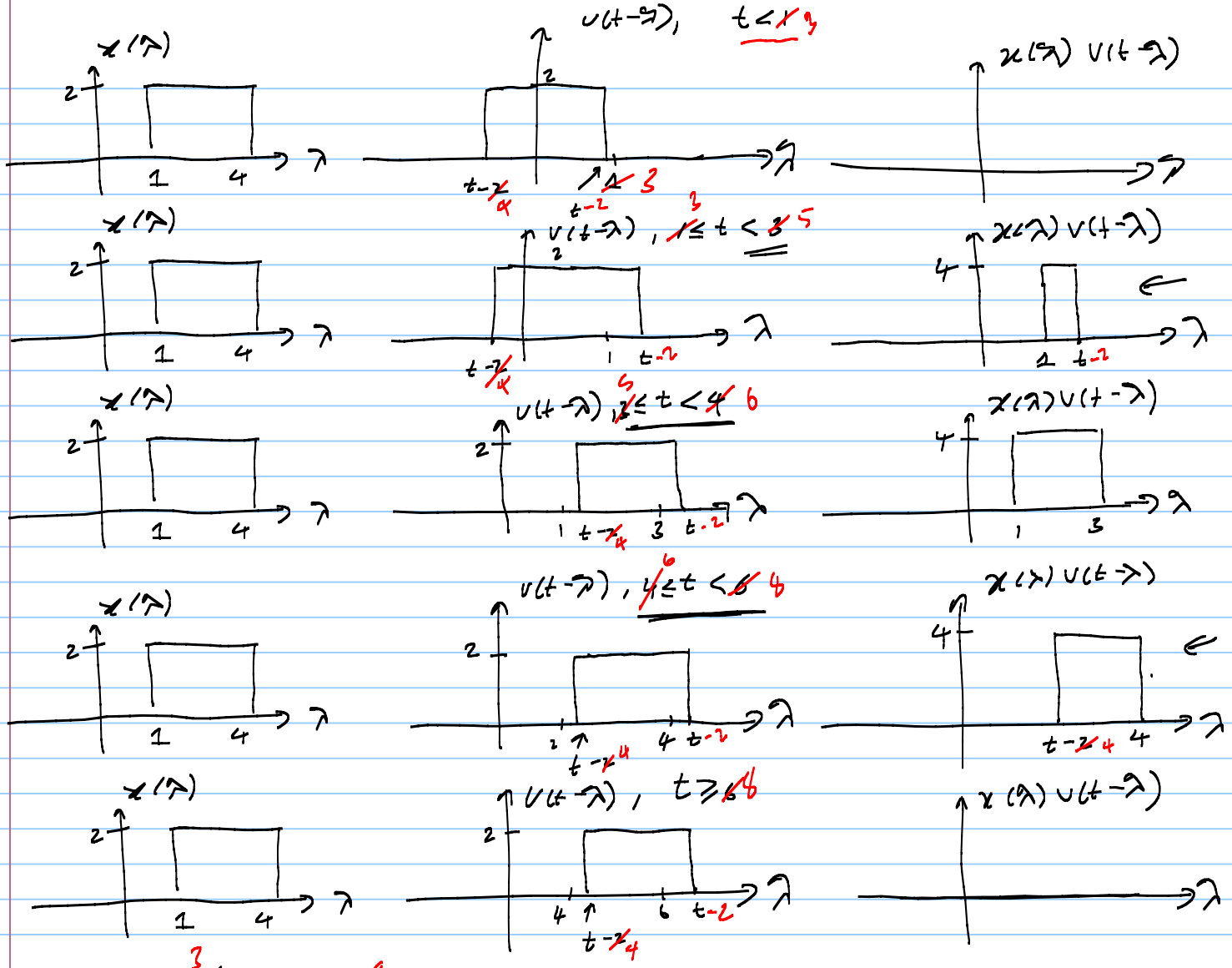
EX 2





$$x(t) * v(t) = ? = \int_{-\infty}^{\infty} x(\lambda)\, v(t-\lambda)\, d\lambda = \int_{-\infty}^{\infty} x(t-\lambda)\, v(\lambda)\, d\lambda$$





For $t < 1$ & $t \geq 6$, $x(t) * v(t) = 0$

For $1 \leq t < 3$

$$x(t) * v(t) = \int_1^{t-2} 4\,d\lambda = 4\lambda \Big|_1^{t-2} = 4t - 4 = 4(t-2) - 4 = 4t - 12$$

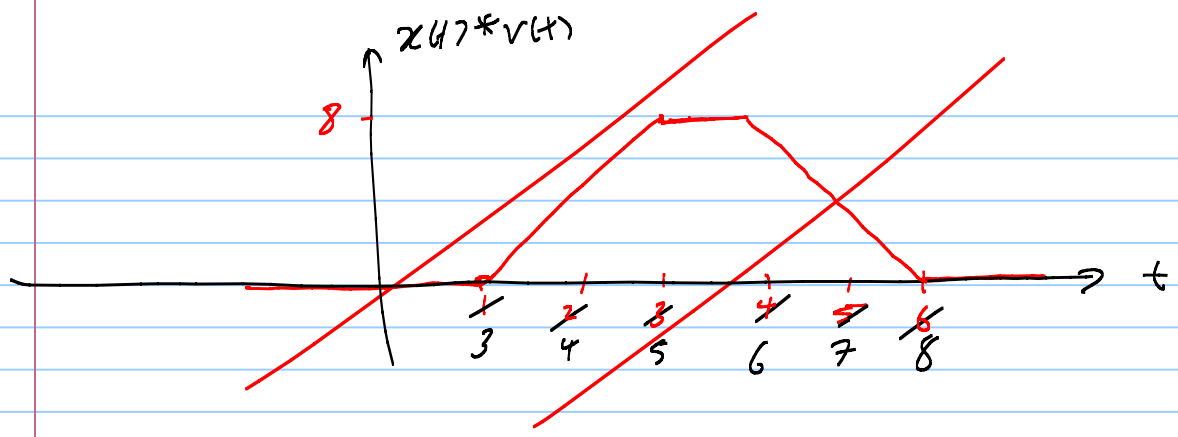
For $3 \leq t < 4$

$$x(t) * v(t) = \int_1^3 4\,d\lambda = 4\lambda \Big|_1^3 = 12 - 4 = 8$$

For $4 \leq t < 6$

$$x(t) * v(t) = \int_{t-2}^4 4\,d\lambda = 4\lambda \Big|_{t-4}^4 = 16 - 4(t-2) = -4t + 24$$

$$= 16 - 4(t-4) = -4t + 32$$



For $3 \le t < 5$

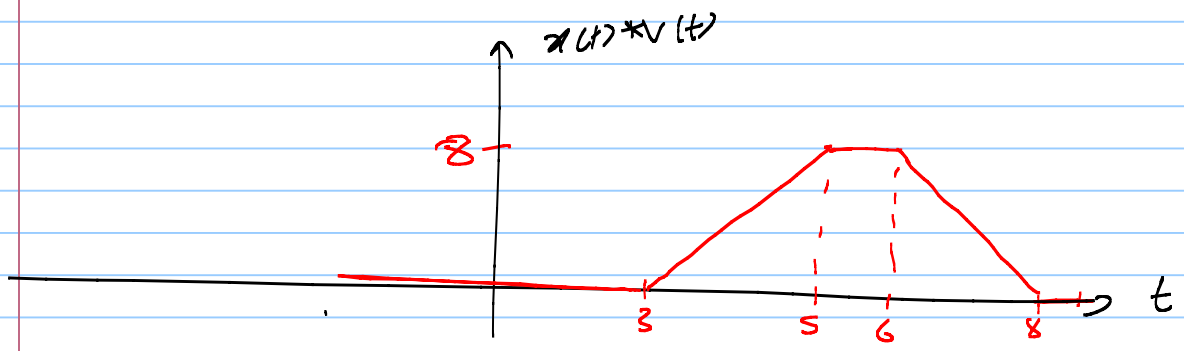
$x(t) * v(t) = 4t - 12$

For $5 \le t < 6$

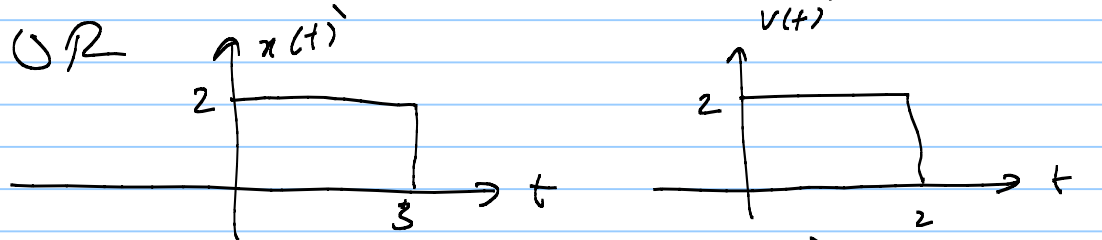
$x(t) * v(t) = 8$

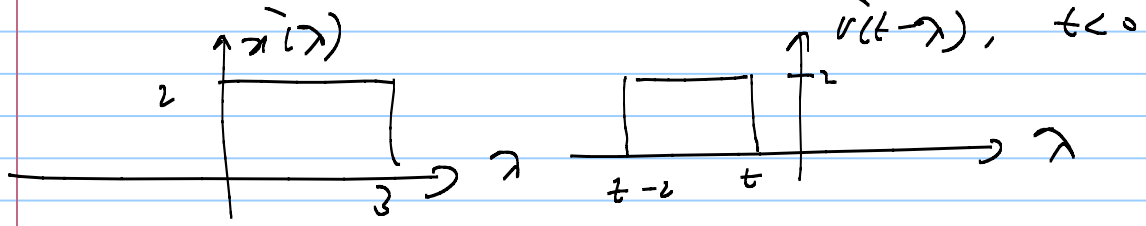
For $6 \le t < 8$

$x(t) * v(t) = -4t + 32$

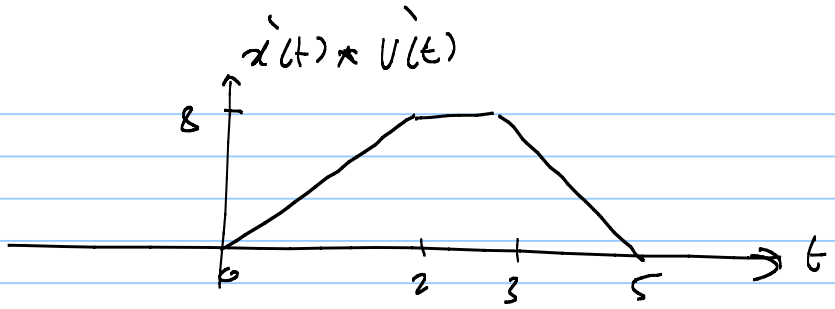


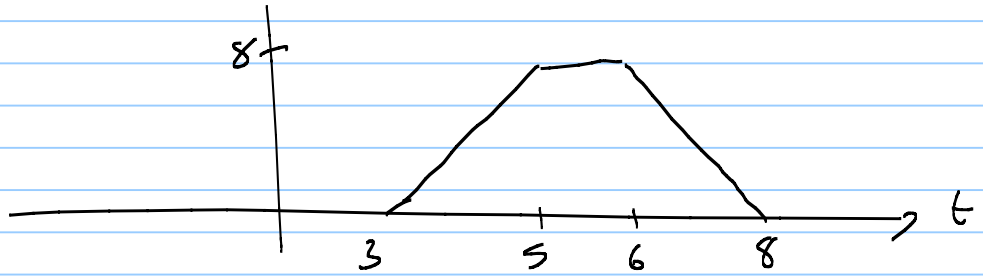
OR

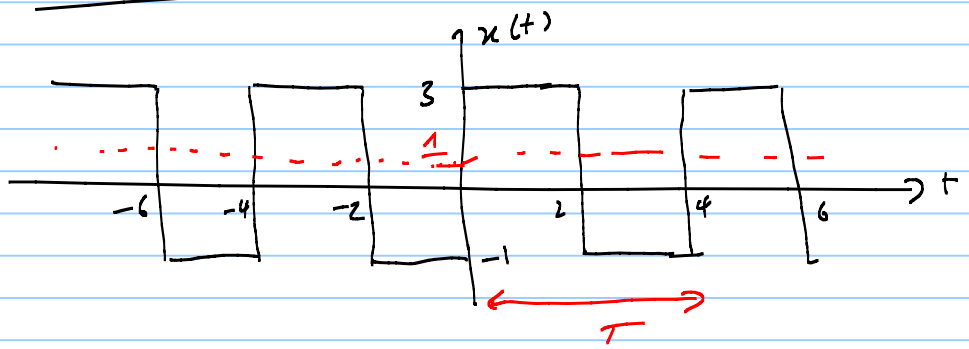




$\int$



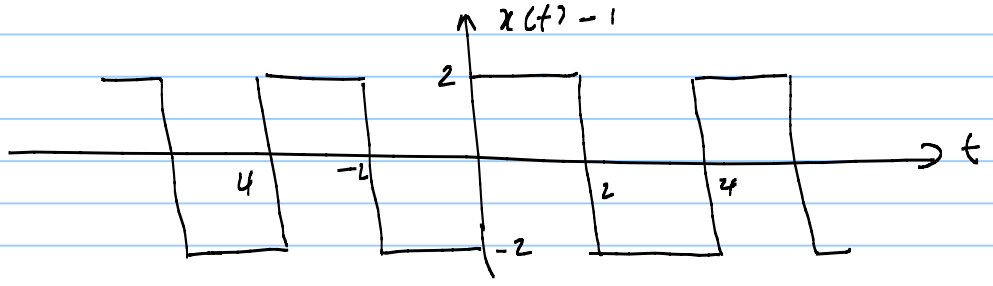




$a_0 = 1$

$T = 4$

$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$



For $x(t) - 1$: $\omega_0 = \frac{\pi}{2}$; $a_0 = 0$; $a_k = 0$

$$b_k = \frac{T}{4}\int_0^{T/2} x(t)\sin(k\omega_0 t)\,dt = \frac{4}{4}\int_0^2 (2)\sin\left(k\tfrac{\pi}{2}t\right)dt$$

$$= -2\cos\left(\tfrac{k\pi}{2}t\right)\Big/\left(\tfrac{k\pi}{2}\right)\Big|_0^2$$

$$= \frac{-4}{k\pi}\cos\left(\tfrac{k\pi}{2}t\right)\Big|_0^2 = \frac{-4}{k\pi}\left[\cos(k\pi) - 1\right]$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

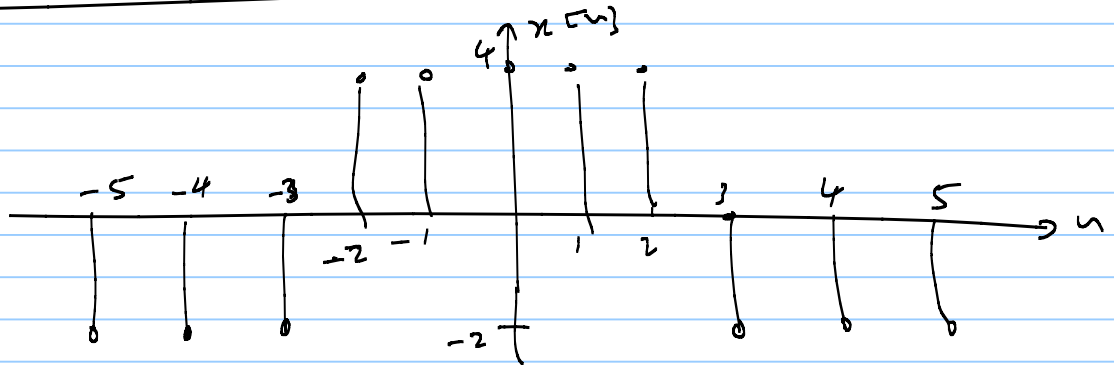
$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\therefore\ b_k = \begin{cases} \frac{8}{k\pi}, & k = 1, 3, 5, 7, \cdots \\ 0, & \text{o.w.} \end{cases}$$

$$x(t) - 1 = \sum_{k\ \text{odd}} \frac{8}{k\pi} \sin\left(k\tfrac{\pi}{2}t\right), \quad -\infty < t < \infty$$

$$\therefore\ x(t) = 1 + \sum_{k\ \text{odd}}^{\infty} \frac{8}{k\pi} \sin\left(k\tfrac{\pi}{2}t\right); \quad -\infty < t < \infty$$

OR $$x(t) = 1 - \sum_{k=0}^{\infty} \frac{4}{k\pi}\left[\cos(k\pi) - 1\right] \sin\left(k\tfrac{\pi}{2}t\right)$$



$$x[n] = -2\,P_2[n+4] + 4\,P_4[n] - 2\,P_2[n-4]$$

$$P_{2q}[n] \leftrightarrow \frac{\sin\left((q+\frac{1}{2})\Omega\right)}{\sin(\Omega/2)}$$

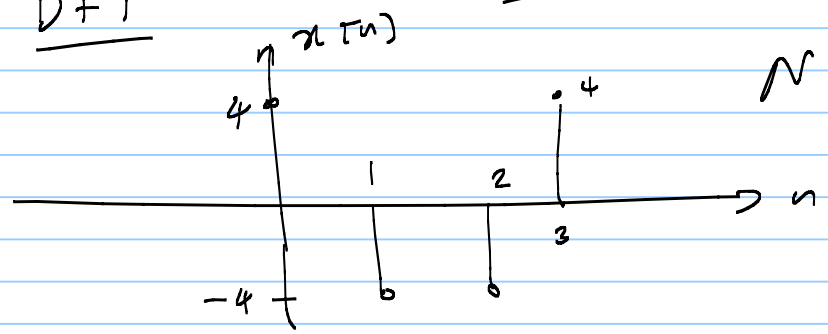
$$X(\Omega) = -2\,\frac{\sin(1.5\Omega)}{\sin(\Omega/2)}\,e^{j4\Omega} + 4\,\frac{\sin(2.5\Omega)}{\sin(\Omega/2)} - 2\,\frac{\sin(1.5\Omega)}{\sin(\Omega/2)}\,e^{-j4\Omega}$$

$$X(\Omega) = \frac{1}{\sin(\Omega/2)}\left[4\sin(2.5\Omega) - 2\sin(1.5\Omega)\left[\frac{e^{j4\Omega} + e^{-j4\Omega}}{2}\right]\right]$$

$$= \frac{4}{\sin(\Omega/2)}\left[\sin(2.5\Omega) - \sin(1.5\Omega)\cos(4\Omega)\right]$$

EX DFT

Four point



$N = 4$

$$X_k = \sum_{n=0}^{N-1} x[n]\, e^{\frac{-j2\pi kn}{N}}$$

$$X_k = \sum_{n=0}^{3} x[n]\, e^{-j k\frac{\pi}{2} n}$$

$$X_k = x[0]\, e^{0} + x[1]\, e^{-jk\frac{\pi}{2}} + x[2]\, e^{-jk\pi} + x[3]\, e^{-jk\frac{3\pi}{2}}$$

$$\Rightarrow X_k = 4 - 4e^{-jk\frac{\pi}{2}} - 4e^{-jk\pi} + 4e^{-jk\frac{3\pi}{2}}$$

$e^{j\frac{\pi}{2}} = \cos(\frac{\pi}{2}) + j\sin(\frac{\pi}{2}) = j$

$e^{-j\frac{\pi}{2}} = -j$

$e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1$

$e^{-j\pi} = -1$

$e^{j\frac{3\pi}{2}} = \cos(\frac{3\pi}{2}) + j\sin(\frac{3\pi}{2}) = 0 - j$

$e^{-j\frac{3\pi}{2}} = j$

$X_0 = 0$

$X_1 = 4 - 4e^{-j\frac{\pi}{2}} - 4e^{-j\pi} + 4e^{-j\frac{3\pi}{2}}$

$X_1 = 4 - 4(-j) - 4(-1) + 4(j)$

$X_1 = 8 + j8$

$X_2 = 4 - 4e^{-j\pi} - 4e^{-j2\pi} + 4e^{-j3\pi}$

$= 4 - 4(-1) - 4(1) + 4(-1) = 0$

$X_3 = 4 - 4e^{-j\frac{3\pi}{2}} - 4e^{-j3\pi} + 4e^{-j\frac{9\pi}{2}}$

$= 4 - 4(j) - 4(-1) + 4(-j)$

$= 8 - j8$

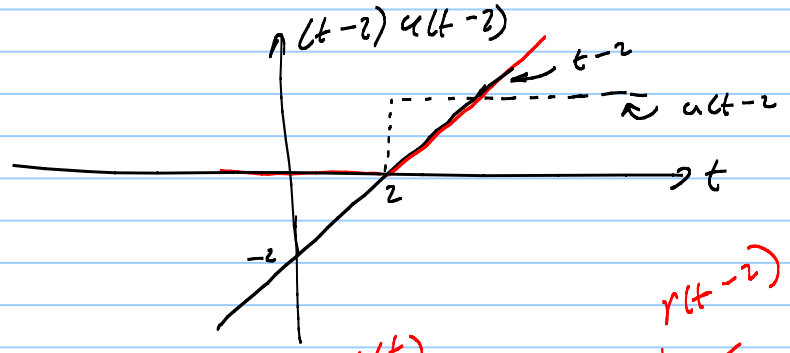
$X_0 = 0$

$X_1 = 8 + j8$

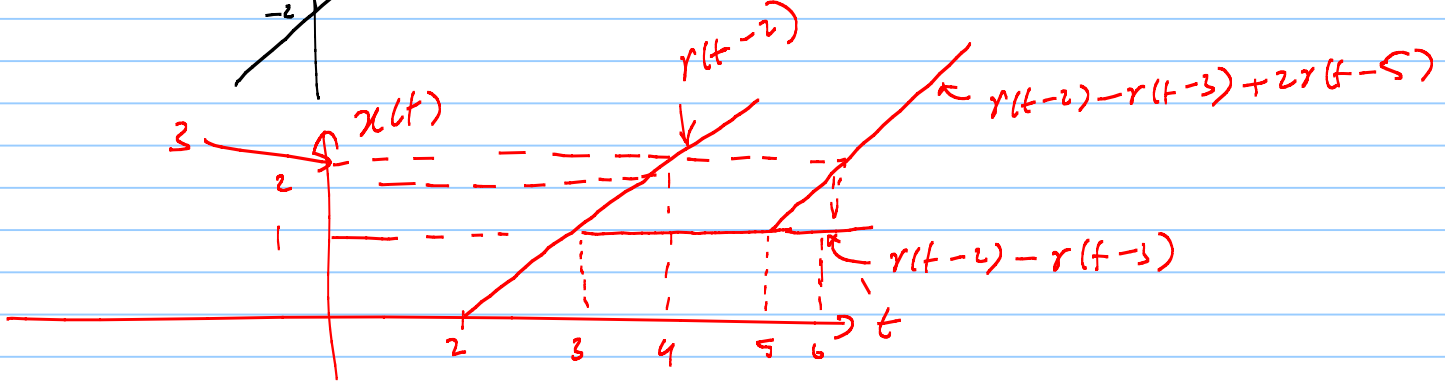
$X_2 = 0$

$X_3 = 8 - j8$

$$x(t) = (t-2)\,u(t-2) - r(t-3) + 2\,r(t-5)$$



$\therefore (t-2)u(t-2) = r(t-2)$



If $X(z) = \ln\left(\frac{4z+1}{5z+2}\right)$; find $x[n]$

$$n\,x[n] \longleftrightarrow -z\frac{d}{dz}\left[\ln\left(\frac{4z+1}{5z+2}\right)\right]$$

$$\frac{d}{dz}\ln(f(z)) = \ln(f(z))'\,f(z)'$$

Let $f(z) = \frac{4z+1}{5z+2} \Rightarrow f(z)' = \frac{(5z+2)(4)-(4z+1)(5)}{(5z+2)^2} = \frac{20z+8-20z-5}{(5z+2)^2}$

$$= \frac{3}{(5z+2)^2}$$

$$\ln(f(z))' = \frac{1}{f(z)}$$

$$\therefore \frac{d}{dz}\ln\left(\frac{4z+1}{5z+2}\right) = \left(\frac{5z+2}{4z+1}\right)\left(\frac{3}{(5z+2)^2}\right)$$

$$\therefore n\,x[n] \longleftrightarrow \frac{-3z}{(4z+1)(5z+2)} = \frac{-3z/20}{(z+0.25)(z+0.4)}$$

$$n x[n] \longleftrightarrow \frac{A}{z+0.25} + \frac{B}{z+0.4}$$

$$A = \left.\frac{-3z/20}{z+0.4}\right|_{z=-0.25} = \frac{0.0375}{0.15} = 0.25$$

$$B = \left.\frac{-3z/20}{z+0.25}\right|_{z=-0.4} = \frac{0.06}{-0.15} = -0.4$$

$$\therefore n x[n] \longleftrightarrow \frac{0.25}{z+0.25} - \frac{0.4}{z+0.4}$$

$$\therefore n x[n] = 0.25(-0.25)^{n-1} u[n-1] - 0.4(-0.4)^{n-1} u[n-1]$$

$$= -(-0.25)^{n} u[n-1] + (-0.4)^{n} u[n-1]$$

$$\therefore \boxed{x[n] = -\frac{(0.25)^n}{n} u[n-1] + \frac{(-0.4)^n}{n} u[n-1]}$$